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Basic Mathematical Investigations in Electromagnetic Wave Theory

Joseph B. Keller

Courant Institute of
Mathematical Sciences
Division of Electromagnetic Research

251 Mercer Street
New York, New York 10012

Contract No. AF 19(628)3868
Project No. 5635
Task No. 563501
Work Unit No. 56350101

Final Report
January 1, 1964—December 31, 1968

January 1969

Contract Monitor: Ronald G. Newburgh
Microwave Physics Laboratory

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Prepared for

Air Force Cambridge Research Laboratories
Office of Aerospace Research
United States Air Force
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Abstract

This report is a summary of research activities pursued during the period from January 2, 1964 to January 1, 1969 by the Division of Electromagnetic Research of the Courant Institute of Mathematical Sciences at New York University, under Contract Number AF 19(628)3868 with the Air Force Cambridge Research Laboratories. The research program was a continuation of work done under Contract Number AF 19(604)5238 with the same agency.

The object of the research was to extend existing knowledge of classical electromagnetic theory with emphasis on electromagnetic wave-guiding radiation and communication, and diffraction problems.

Investigations have been conducted in various areas in accordance with these objectives and are explained in the body of this report.

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Introduction

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During the contract period, a total of 30 reports have been issued 25 of which have been published or accepted for publication in technical journals.

We have been fortunate in having some distinguished guests speak at our weekly Electromagnetic Theory Seminars and have listed them at the end of this report.

A complete list of reports begins on page 38.

I. Diffraction Theory

1. Propagation in inhomogeneous and anisotropic media. Time-dependent problems.

Maxwell's equations in isotropic and anisotropic media, the linearized equations of compressible fluid dynamics and many of basic linearized systems of plasma physics have in common the property of being expressible as a first-order symmetric-hyperbolic system of partial equations. In three space variables, $X = (x_1, x_2 x_3)$, these equations have the form

$$A_o \partial_t U + A_1 \partial_{x_1} U + A_2 \partial_{x_2} U + A_3 \partial_{x_3} U + \lambda BU = f(t,X,\lambda) \tag{1}$$

with initial conditions

$$U(0,X;\lambda) = U_o(X;\lambda). \tag{2}$$

Here, A_o, A_1, A_2, A_3 and B are real NxN matrices which are functions of X, A_o is positive definite and A_1, A_2 and A_3 are symmétrie. U denotes an N-component column vector. λ denotes a parameter which is to be regarded as fixed until further notice

In general, even for the case of homogeneous media [constant A_ν, $\nu = 0,\cdots,3$], and then only for very special geometries, such systems are not rigorously solvable by the standard techniques of mathematical physics. This circumstance has inspired, in recent years, the invention of asymptotic methods which, though approximate and formal, yield physically interesting and useful results for a large class of initial-value and boundary-value problems.

One such method is the so-called progressing wave formalism

[J.B. Keller [1]-[6], D. Ludwig [7a, 7b], P.D. Lax [8], F.G. Friedlander [9], among others]. By inserting a series expansion of the form

$$U(t,X) = \sum_{m=o}^{\infty} e_m(\phi(t,X))Z^{(m)}(t,X) \tag{3}$$

into (1), differential equations for the phase function $\phi(t,X)$ and amplitude factors $Z^{(m)}$ are obtained. These equations can be solved with the aid of ordinary differential equations, the so-called ray equations, which yield as solutions curves in space-time called bicharacteristics. The bicharacteristics fill out characteristic surfaces of the equation (1) in space-time. The function $e_o(\tau)$ $(m = 1,2,\ldots)$ is arbitrary---it may even be a distribution. But for $m \geq 1$, $e_m(\tau)$ is required to satisfy $e'_m(\tau) = e_{m-1}(\tau)$.

The interpretation of the progressing wave expansion (3) depends upon the choice of the 'wave form' $e_o(\tau)$. If $e_o(\tau) = \exp\{-i\omega_o\tau\}$ and $\phi(t,X) = t-\psi(X)$ we are led to the standard asymptotic theory of time-reduced system

$$\sum_{\nu=1}^{3} A_\nu \partial_{x_\nu} U + (\lambda B - i\omega_o A_o)U = 0, \tag{4}$$

[cf. [1]], where ω_o is regarded as a large parameter. More generally, if we set $e_o(\tau) = a(\tau)\exp(-i\omega_o\tau)$ where again ω_o is large we obtain an asymptotic theory of high-frequency amplitude modulated waves. If $e_o(\tau)$ is discontinuous or even a distribution the functions $e_m(\tau)$ are successively smoother and the expansion (4) can be used to reduce problems with singular initial data to problems with smooth data. In this case, the progressing wave formalism becomes a theory of propagation of singularities. For example, if $e_o(\tau)$ vanishes for $\tau < 0$, the successive

integrals $e_m(\tau)$ also vanish for $\tau < 0$ and it can be shown [D. Ludwig [3]] that the expansion (3) converges in a sufficiently small neighborhood of $\phi = 0$. In this case the expansion (3) correctly describes the behavior of solutions of (1) in a small region just behind the advancing front.

In EM-196 (Lewis), a detailed study is made of the progressing wave formalism and the results are applied to the study of the important problem of radiation from moving sources and that of reflection by moving surfaces.

One of the defects of the progressing wave formalism is its failure to account for dispersion. In mathematical terms this results [restricting ourselves for the moment to the familiar time-reduced system (4)] from the fact that the undifferentiated term λBU in (4) is regarded as small in comparison with terms that are multiplied by the large parameter ω_o. A new asymptotic theory is necessary when this approximation is no longer valid. In EM-197 Dr. R.M. Lewis has given such theory. This asymptotic method gives results which differ from the corresponding results for the progressing wave formalism despite the fact that most of the formal techniques used in both theories are the same. Thus, in the new theory with dispersive terms taken into account the 'ray equations' yield 'rays' which do not fill out characteristic surfaces of the equation. They represent rather the trajectories of points which move with an appropriately defined group velocity.

An application of the theory developed by R.M. Lewis in EM-197 to electromagnetic wave propagation in an anisotropic medium is given in EM-232 (Lewis and Granoff).

Specifically, the authors show how the time-dependent magnetoionic equations can be filled into the framework of the theory of dispersive equations discussed above in the cases where the gyro frequency alone or the gyro frequency and plasma frequency are large. The interesting result is a formula for the rate of rotation of the electric vector about a curved ray in an inhomogeneous medium which reduces to that of the well-known Faraday rotation for the special case of a plane wave in a homogeneous medium.

This paper was presented by R.M. Lewis at the U.R.S.I. Symposium on Electromagnetic Waves. Stresa, Italy, June 1968.

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5. J.B. Keller, R.M. Lewis, and B.D. Seckler, "Asymptotic Solution of Some Diffraction Problems," Comm. Pure Appl. Math., Vol. 9, p. 207 (1956).

6. R.M. Lewis and J.B. Keller, "Asymptotic Methods for Partial Differential Equations: The Reduced Wave Equation and Maxwell's Equations," NYU Research Report, E.M.-194 (1964).

7a. D. Ludwig, "Exact and Asymptotic Solutions of the Cauchy Problem," Comm. Pure Appl. Math., Vol. 13, p. 473 (1960).

7b. D. Ludwig, "The Singularities of the Riemann Function," NYU Research Report, A.E.C., N.Y.O.-9351.

8. P.D. Lax, "Asymptotic Solutions of Oscillatory Initial Vlue Problems," Duke Math. J., Vol. 24, p. 627 (1957).

9. F.G. Friedlander, Sound Pulses (London: Cambridge University Press, 1958).

In EM-196 (Lewis), the progressing wave formalism is a generalization of geometrical optics which provides a method for obtaining approximate solutions of initial-boundary value problems for the wave equation and other linear hyperbolic equations. The formalism includes, as a special case, the asymptotic (high frequency) theory of the reduced wave equation and related equations. However, the progressing wave formalism is not limited to the analysis of time-harmonic fields. It provides an approach to the study of amplitude-modulated waves, the propagation of discontinuities and other singularities, and the behaviour of arbitrary waves in the neighborhood of a wave front. In addition, it is capable of treating reflection by moving surfaces and radiation from moving sources.

EM-197 (Lewis) deals with a general method for finding asymptotic solutions of initial-boundary value problems for linear hyperbolic partial differential equations. A large parameter λ appears in the equation, multiplying a lower order (dispersive) term. It also may appear in the initial data and the inhomogeneous (source) term, in a variety of ways. This gives rise to a variety of different types of asymptotic solutions. The expansion procedure is a "ray method," i.e., all of the functions that appear in the expansion satisfy ordinary differential equations along certain space-time curves called rays. These rays do not lie on characteristic hypersurfaces, but instead fill out the interior of the characteristic hypercone. They are associated with an appropri-

ately defined group velocity. The details of the expansion are presented for a simple second order hyperbolic equation. The applicability of the method to symmetric hyperbolic systems and to the integro-defferential equations for dispersive dielectrics is discussed.

EM-232 (Lewis and Granoff) deals with recently developed "ray methods" for the asymptotic solution of linear partial differential equations applied to a system of time-dependent magneto-ionic equations. These are equations for E, H, and the electron current, J. For time-harmonic waves J can be eliminated and the equations reduce to the usual equations of the magneto-ionic theory. The time-dependent equations contain a large parameter λ, the plasma frequency, which serves as an expansion parameter. In the "strongly anisotropic" case both the gyro frequency and plasma frequency are large. In the simpler "weakly anisotropic" case only the plasma frequency is large. Both cases can be treated by the general method presented. The weakly anisotropic case is analyzed in greater detail. One result of this analysis is a formula for the rate of rotation of the electric vector about a curved ray in an inhomogeneous medium. The formula reduces to that of the well-known Faraday rotation for the special case of a plane wave propagating in a homogeneous medium.

2. Diffraction by smooth bodies and edges.

Work on diffraction by smooth bodies and edges has been largely based upon Keller's geometrical theory of diffraction, and various extensions of Keller's theory. The subject as it existed in 1964 is summarized in EM-194 (Lewis and Keller). This report is a forerunner of their forthcoming monograph on asymptotic theory of wave propagation.

Work since that time has emphasized behavior in transition regions, which connect regions where the solution has different asymptotic behavior for high frequencies. For example, the penumbra or shadow transition region connects a region where ordinary geometrical optics is valid with a region where some form of diffraction theory must be used. The behavior in transition regions is of importance for its own sake; for example, behavior in the penumbra is important for radio communication. In addition, uniform asymptotic expansions valid in the transition regions often lead to improvements of the simpler geometrical theory, to yield higher terms in the expansions and to permit extension to inhomogeneous and anisotropic media.

The most remarkable analysis of transition regions is given in EM-203 (Nussenzveig), which treats high-frequency scattering by an impenetrable sphere. There ten different regions are given, in which different asymptotic forms of the solution are valid. The more general problem of diffraction by a convex object is treated in EM-217 (Ludwig), based upon a uniform asymptotic expansion of the field at a caustic. Similar ideas are used to extend and complete Keller's theory of creeping waves in EM-219 (Lewis, Bleistein and Ludwig). The transition through the penumbra region (which connects creeping waves and the "flat earth"

approximation) is described in EM-225 (Rulf). A similar transition between creeping waves and lateral waves at a curved interface is given in EM-221 (Rulf).

Keller's geometrical theory of edge diffraction is extended to a uniform theory in EM-220 (Lewis and Boersma), using the exact solution for diffraction by a half-space as a first approximation. The geometrical theory of edge diffraction is used to study modes in an open-ended wave guide in EM-229 (Yee, Felson and Keller).

Finally, the problem of diffraction of electromagnetic waves by a circular aperture in an infinitely conducting plane screen is studied in EM-195 (Bazer and Rubenfeld). This work is set apart from the other works reported on here in its restriction to low frequencies, and its use of power series expansions and integral equations.

Each of the preceding reports is described in more detail in the following:

The theory of asymptotic solutions of the wave equation and of Maxwell's equations is summarized in EM-194 (Lewis and Keller). The ideas of classical geometrical optics are used to construct approximate solutions through introduction of wave fronts, rays and transport equations. The problems of radiation from point or line sources, reflection at a boundary and reflection and transmission at an interface can be treated by these methods. Keller's geometrical diffraction theory can then be applied to describe diffraction by edges and vertices, smooth body diffraction, and propagation in stratified media.

In EM-203 (Nussenzveig), the scattering of a scalar plane wave by a totally reflecting sphere (hard-core potential) at high frequencies

is treated by a modified Watson transformation. The behavior of the solution both in the near and far regions of space is discussed, as well as the accuracy and domain of applicability of the geometrical optics approximation and classical diffraction theory. It is shown that different transformations are required in the forward and backward half-spaces, and corresponding integral representations for the primary wave are derived. The transformations are rigorously proved and the convergence of the residue series is discussed. In the shadow region, the physical interpretation of the complex angular momentum poles in terms of surface waves is in agreement with Keller's geometrical theory of diffraction. In the lit region, sufficiently far from the shadow boundary, the geometrical optics expansion for the wave function is confirmed up to the second order. On the surface of the sphere, Kirchhoff's approximation is accurate, except in the penumbra region, where the behavior is described by Fock's function. The diffraction effects in the neighborhood of the shadow boundary are investigated and the corrections to classical diffraction theory are obtained. The shift of the shadow boundary is evaluated. The expression for the wave function in the Fresnel-Lommel region is derived and applied to the discussion of the Poisson spot and the behavior near the axis. The total scattering amplitude is evaluated for all angles, including the neighborhood of the forward and backward directions. The corrections to the forward diffraction peak and the transition to the region of geometrical reflection are discussed. The modified Watson transformation is also applied directly to the scattering amplitude. The connection between representations valid in different regions is established.

In EM-217 (Ludwig), a uniform asymptotic expansion of the field scattered by a general convex body is given. The asymptotic solution has a complicated form, analogous to an integral representation of the field scattered by a circular cylinder. In fact, the solution is constructed by introducing generalized Bessel and Hankel functions. In the deeply illuminated region, the solution can be simplified, and the reflected field is obtained. In fact, this construction leads to a proof of the validity of geometrical optics in this region. In the deep shadow the solution can be simplified to yield the creeping waves predicted by Keller's geometrical theory of diffraction.

Report EM-219 (Lewis, Bleistein and Ludwig) is a closely related work which treats creeping waves in more detail. Part of Keller's theory may be deduced from a formal series involving an exponential function. The unknown phase and amplitude functions which appear in the series can be determined by solving ordinary differential equations along certain curves called "rays." This "deductive part" of Keller's theory does not suffice to solve most diffraction problems asymptotically. It must be supplemented by "non-deductive" parts of the theory which are based in part on physical intuition and lead to the concepts of "surface rays," "diffraction coefficients," "decay exponents," etc.

In EM-219 the non-deductive parts of Keller's theory are almost entirely eliminated, for problems involving smooth convex obstacles. This is achieved by means of a formal series involving the Airy function. The unknown phase and amplitude functions in this series are determined much the same way as in Keller's theory. In contrast to the latter, the new expansion remains valid at the surface of the obstacle

which is a caustic of the diffracted rays. Away from the surface the new expansion reduces to Keller's result. Now the diffraction coefficients and decay exponents are obtained deductively without the use of "canonical problems." For the case of an inhomogeneous medium some minor errors in Keller's results are corrected.

Report EM-225 (Rulf) is concerned with the penumbra region. The main contribution to the field of a radiating antenna in the penumbra region (i.e. beyond the horizon but not "very far" in the shadow region) is due to diffraction of the ground wave around the earth's surface. The model of a source near a sphere (either dielectric or with an impedance surface) has been used to investigate this problem. The difficulties to extract numerical information from available exact solutions are the following: the "creeping wave" series, which converges very rapidly in the deep shadow region, converges rather slowly in the penumbra region. The "flat earth" model on the other hand neglects the important effect of the earth's curvature. An attempt to close the gap between these two approximations was made by Wait. In this work a slightly different asymptotic expansion of the field is given. This result is valid only in the penumbra region, with the penumbra clearly defined. It is also shown how the result is related to, and reduces to, the "creeping wave" series in the deep shadow region.

In EM-221 (Rulf), diffraction effects at a gently curved interface between two media are investigated. Particular attention is paid to the behavior of the field on the diffracted rays which propagate along the interface into the shadows. It is found that far from the launching point of such a ray the field comprises of a series of modes

which decay exponentially, due to the continuous leakage of energy away from the interface. At moderate distances, in the penumbra region, this series is poorly convergent. It can be converted into an integral, which can be evaluated asymptotically there, yielding a field with an algebraic decay. The field is like that diffracted along a plane interface, the so-called lateral wave, and reduces to it when the radius of curvature becomes infinite. The regions of transition from one representation to the other are determined, and uniform asymptotic expressions, valid across those regions, are given. All of the results apply to a two-dimensional scalar problem, but results for three dimensions and for vector problems can be derived in a similar way.

Report No. EM-220 (Lewis and Boersma), gives a uniform asymptotic theory of edge diffraction. Geometrical optics fails to account for the phenomenon of diffraction, i.e., the existence of non-zero fields in the geometrical shadow. J.B. Keller's geometrical theory of diffraction accounts for this phenomenon by providing correction terms to the geometrical optics field, in the form of a high frequency asymptotic expansion. In problems involving screens and on shadow boundaries where the expansion has singularities. The uniform asymptotic theory presented in EM-220 provides a new asymptotic solution of the diffraction problem which is uniformly valid near edges and shadow boundaries. Away from these regions the solution reduces to Keller's theory. However, singularities at any caustics other than the edge are not corrected.

In EM-229 (Yee, Felsen and Keller), an open-ended waveguide is treated by ray methods. Ray methods have been widely used to calculate high frequency scattering by objects in an unbounded medium, but they have not been used much for waveguide problems. To show that they can be used for waveguides, ray methods are used to treat reflection from an open-ended parallel plane waveguide propagating several modes. The incident mode is decomposed into two plane waves whose scattering by the edges at the termination produces the reflected field. The singly diffracted cylindrical wave originating at each edge, which is known from the asymptotic theory of diffraction by a single wedge, is represented by means of diffracted rays. Then the sum of the fields on the multiply reflected rays is converted into modal form. This yields formulas for the reflection coefficients in the various modes due to single diffraction. In addition double and multiple diffraction are also taken into account, yielding improved formulas for the reflection coefficients. The results of extensive numerical calculations for TE and TM modes incident in flanged and unflanged open-ended waveguides are presented, and are compared with calculations based on exact solutions when these are available. It is found that the single diffraction results account for the average behavior of the amplitude and phase of the reflection coefficients. The inclusion of multiple diffraction accounts for the finer details and yields excellent accuracy even for waveguide height-to-wavelength ratios as small as 1/3 in the TM case. These results suggest that the ray method also provides an accurate means for analyzing other types of waveguide discontinuities.

In EM-195 (Bazer and Rubenfeld), the problem of the diffraction of electromagnetic waves by a circular aperture in an infinitely conducting planar screen is studied. Attention is focused chiefly on the low frequency end of the spectrum where the product ka of the wave number k and the aperture radius a is small. Transverse electric and transverse magnetic modal excitation appropriate to the cylindrical geometry of the problem is assumed. New integral representations of the solution corresponding to incident fields of this kind are presented. These representations, which involve a pair of unknown functions, are designed so as to automatically satisfy Maxwell's equations, the boundary conditions, the radiation condition and all but one of the edge conditions. The unknown functions are shown to satisfy a pair of integro-differential equations which follow directly from the representations by means of an elementary calculation. Convergent power series solutions of these integro-differential equations are readily obtained for small enough ka, when it is assumed merely that the excitation is sufficiently regular in the aperture. These results are employed to derive the first terms in the power series expansions of the far fields, the edge fields, the aperture fields and of the transmission coefficients. When the incident transverse electric and magnetic modes are specialized to those which occur in plane wave excitation these expressions are found to reduce to the well-known plane wave formulas obtained by other means.

3a. Inverse problems

In EM-218 (Keller), several inverse scattering problems are formulated. The questions of existence and uniqueness of solutions of these problems are touched on briefly. Then various methods for obtaining approximate solutions of the direct scattering problem are described. These include the methods of geometrical optics and physical optics, the geometrical theory of diffraction and the quasi-static method. Each such method gives rise to a particular formulation of the inverse scattering problem. Some of the methods for solving these problems are presented, together with the results obtained with them.

Details are given with respect to the geometrical optics method and the physical optics method. The latter method is exploited in two ways. The first way yields an approximation to the shape of a body of revolution in terms of the pattern function for a fixed frequency and all angles. The second way gives an expression for the lengthwise distribution of crossectional area of a body of revolution in terms of the frequency dependence of the back scattered amplitude.

3b. Far field expansions The concept of a plane wave is widely used in electromagnetic theory. Actually such a plane wave is an idealization of a finite distant segment of a radiation pattern, if intuition is any guide. In fact, the radiation from a collection of cylindrical sources within a bounded cylindrical domain, when viewed by an observer at an infinite distance, cannot be distinguished from a plane wave. As the observer approaches closer to the source region, the nature of the radiation may appear cylindrical or perhaps even more complicated. If, however, the distance from the source is sufficiently

large, it may be said that the wave is almost, but not quite a plane wave.

The notation described above is treated mathematically in EM-199 (Karp& Zitron). This paper is concerned with the relation between a general cylindrically radiated wave and a plane wave. It is shown here that the relation is expressible as an asymptotic series involving plane waves and their derivatives of various orders with respect to angle of incidence.

An asymptotic expansion for two-dimensional outwardly radiating fields is developed from an integral representation of these fields by means of a saddle point integration. The expansion is given in terms of inverse powers of the distance between the origins of coordinate systems used to describe fixed source region and fixed evaluation region. The coefficients are expressed in terms of plane waves and linear combinations of derivatives of plane waves with respect to angle of incidence. The theorem may be employed in scattering problems in reducing scattering of arbitary two-dimensional fields by arbitrary cylinders to scattering of plane waves by the arbitrary cylinders.

II. Surface Waves

It is well known that surface waves can be excited and propagated along particular idealized electromagnetic structures. Examples are dielectric coated surfaces, dielectric slabs and corrugated or laminated structures. In so far as the surface wave feature is concerned, it is known that the details of the structure may be suppressed and replaced by an impedance boundary condition. Subsequent developments showed that the use of impedance boundary conditions facilitated the solution of more complex problems such as the excitation, propagation and diffraction of surface waves on structures with discontinuities in impedance and geometry. The phenomenological representation by an impedance boundary condition is also useful in analyzing the effect of other structures such as absorbers and surfaces with anisotropic conductivity.

Since an open surface structure acts to some extent like a waveguide, and since its theory is of use in surface wave antennas, it would seem to be of interest to generalize the existing theory to multimode surface wave guides. This means to generalize the body of theory mentioned above to the case of surfaces that support more than one surface wave. Thus one should ultimately possess a body of theory for surface wave guides analogous to that already available for conventional wave guides. Typical problems of interest are the analysis of the effects of radiation from junctions, changes in cross section, effects of terminations, etc.

In EM-202 (Chu and Kouyoumjian), the special case of one mode was studied with a view to assessing the efficacy of a certain mathematical approximation to the solution of the already approximate problem in the case of a wedge. The approximate method employed is the Kirchoff method.

This paper is an outgrowth of an earlier work by the authors. In two earlier papers the diffraction of a surface wave by a right-angle wedge was treated; one face of this wedge is perfectly conducting and the other is reactive. It was pointed out that the Kirchoff theory of diffraction provides a useful approximation here. A readily calculable result has not been obtained for wedge angles other than 90° and 180°, so the diffraction patterns for various wedge angles are calculated here by the Kirchoff approximation and compared with some measured patterns of a wedge excited by a very narrow H-plane sectorial horn. Both the calculated and measured patterns show that in general the wedge angle has a relatively small effect on the diffraction pattern of a loosly bound surface wave, except when the wedge angle is close to 180°.

In our other papers on this subject we pursued the program outlined in EM-198. The fact that a certain surface supports surface waves (possibly several) is abstracted, and is expressed as a boundary condition of higher order.

The boundary condition we employ to describe the propagation of several surface waves along a plane surface wave structure is given by

$$\prod_{i=1}^{n}\left(\frac{\partial}{\partial y} + \lambda_i\right)u(x,y) = 0, \qquad y = 0 \tag{1.1}$$

where $u(x,y)$ is the z-component of the magnetic vector $\vec{H}(x,y)$, x and y are the usual cartesian rectangular coordinates, and λ_i are constants characteristic of the surface. The value of λ_i is given by

$$\lambda_i = i\omega\varepsilon Z_i = i\omega\varepsilon(R_i - iX_i), \tag{1.2}$$

where ε is the permittivity of free space, ω is the angular frequency, and Z_i, R_i and X_i are the impedances, resistances and reactances of the surface, respectively. For surface wave propagation it is necessary to require that*

$$\mathrm{Re}\lambda_i > 0. \tag{1.3}$$

In EM-198 (Karp and Karal), we give the formal solution for a line source above a plane in the case of n = 3, and give the radiation patterns and surface wave terms in elementary form. A general method is indicated for obtaining the radiation field algebraically for n modes.

The discussion is continued in EM-201, and the possibility of exploitation of complex λ's is pointed out. The method of application to finite structures is indicated. The question of how to assign a finite number of λ's in a representative way for a given surface is considered. The case of a slab of dielectric supplies an illustration of a surface having an infinite number of λ's only a few of which are real. We found that if only a finite number of λ's are used, one does better by not using true impedances.

---

*This assumption is not an essential restriction of the method.

In EM-216 (Karp and Karal), related considerations are undertaken and carried out in detail. The notion of generalized impedance boundary conditions introduced earlier by the authors is discussed and applied to the problem of an infinite dielectric slab lying on a perfect conductor and excited by a TE line source. Since the solution for the dielectric slab is well known, it provides a means to check the number of parameters required to characterize the dielectric slab structure adequately by means of a generalized impedance boundary condition. It is found, for a typical case where one real surface wave mode propagates in the slab, that the dielectric slab structure can be represented quite well by one real and one pair of conjugate complex parameters. Numerical comparisons between the exact solution, and the exact solution for an impedance boundary value problem involving three suitably chosen parameters, are given.

This paper also illustrates the application in practical cases. A kind of inverse problem is involved. The plane wave reflection coefficient is regarded as given as a function of angle. The effective λ's are obtained from this function by numerical methods. The true λ's are infinitely many in number, and are poles of the reflection coefficients.

In practical cases one would only expect to be given the reflection coefficient numerically, graphically, or perhaps statistically.

Hence the determination of a few effective λ's may be more reasonable than the determination of the infinitely many true poles.

In EM-198 (Karp and Karal), the discussion of the behaviour

of the solution for a multiple impedance surface was not carried out, and complex modes were deemphasized. A more complete study was therefore carried out in EM-215 by Morgan, Karal, and Karp. In this paper the phenomenological theory of multimode surface wave excitation is applied to the problem of a line source above an infinite plane exciting surface. Mathematically, this involves solving the reduced wave equation subject to an Nth order differential boundary condition, constructed to allow for the excitation of surface and leaky wave modes. The results of this paper show that the response of any plane surface that is characterizable by such a boundary condition exhibits certain characteristic phenomena. Namely, the far field is composed of a radiated field and excited modes and there exist sectors where the excited modes disappear. Also, it is possible for energy to be transported to infinity along the surface. These phenomena are familiar in the case of a dielectric slab.

The solution is obtained by introducing a suitable auxiliary operator which is then inverted. This allows the determination of the radiated far field by elementary algebraic techniques. Then the result is transformed into a contour integral and the excited modes and high frequency approximation are discussed.

A major purpose of the use of the impedance boundary condition is to facilitate the mathematical analysis of diffraction problems on modified surface structures. This program is pursued in several papers, in the multi-mode case.

In EM-212, the problem of uniqueness is studied by R.C. Morgan. It is well known that the problems of diffraction by mathe-

matically singular surfaces do not possess a unique solution unless special conditions are imposed. For the case of an infinite conducting wedge, there are two singular features of the geometry. On the one hand the wedge extends to infinity; on the other hand the edge presents a discontinuous slope. The former singularity had been attended to by Sommerfeld's radiation condition in a generalized form due to Stoker and Peters*, and the latter by Meixner's edge condition, in the case of Dirichlet and Neuman problems. For the case of one λ, the generalization was given in Morgan and Karp** (1962, 1963) now, however, that the extension to a multi-mode condition was in view, the entire question of uniqueness has to be reconsidered, and this was studied in EM-212 by R.C. Morgan.

Uniqueness is demonstrated for the solution to the reduced wave equation subject to a mixed boundary value condition that excites two surface wave modes. The configuration is taken as a right-angled wedge and the edge condition assumed has the form

$$\sum_{i=o}^{2} \left|\frac{\partial^i u}{\partial x^i}\right| = 0\left(\frac{1}{r^{1+h}}\right), \qquad 0 \leq h < 2/3, \qquad \text{for } r \to 0.$$

It is conjectured that the same procedure may be used to prove uniqueness for the corresponding N-mode problem under the edge condition

$$\sum_{i=o}^{N} \left|\frac{\partial^i u}{\partial x^i}\right| = 0\left(r^{-\left[\frac{(2N-1)}{3} + h\right]}\right), \; 0 \leq h < 2/3, \qquad \text{as } r \to 0.$$

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*Peters, A.S., and Stoker, J.J., A uniqueness theorem and a new solution for Sommerfeld's and other diffraction problems, Comm. Pure Appl. Math., Vol. 7, 1954, pp. 565-585.

**Morgan, R.C., and Karp, S.N., Uniqueness Theorem for a Surface Wave Problem in Electromagnetic Diffraction Theory, Comm. Pure and Appl. Math., Vol. 16, 1963, pp. 45-56. Also, New York Univ., Courant Inst. Math. Sci., Div. of Electromagnetic Res., Res. Rep. No. EM-178, 1962.

The existence of the solutions of the problems whose uniqueness was studied in EM-212 was insured by the study embodied in EM-213, in which the solution is exhibited explicitly for a two-mode problem, under the edge conditions given in EM-212.

Specifically this paper extends the phenomenological theory of multi-mode surface wave diffraction to a right-angled wedge configuration.

The solution to a two-mode problem is obtained under the edge condition

$$\sum_{i=o}^{2} \left|\frac{\partial^i u}{\partial x^i}\right| = O(r^{-[1+h]}), \qquad 0 \leq h < \frac{2}{3}$$

as $r \to 0$. It is conjectured that the same procedure may be used to construct the solution to the corresponding N-mode problem under the edge condition

$$\sum_{i=o}^{N} \left|\frac{\partial^i u}{\partial x^i}\right| = O(r^{-[(2N-1)/3+h]}), \qquad 0 \leq h \leq \frac{2}{3}$$

as $r \to 0$.

Further outgrowths of the effort are given below with the author's abstract. "Uniqueness Theorem for the Reduced Wave Equation under an N-th Order Differential Boundary Condition." Proceedings of the Amer. Math. Society, 17, 4, (1966) pp. 780-787.

Uniqueness is demonstrated for the solution, $u(x,y)$, to a mathematical problem that arises in the phenomenological theory of multi-mode surface wave propagation. The unusual aspect is that a boundary

value problem for the reduced wave equation is posed using the boundary condition

$$\prod_{m=1}^{N}\left(\frac{\partial}{\partial y} + \lambda_m\right)u = 0, \quad y = 0.$$

The solution is assumed to be decomposed into three terms; a source, excited modes (waves of the form $e^{-\lambda_m y + i\sqrt{K^2+\lambda_m^2}}$), and a radiating field. This last term satisfies a Sommerfeld radiation condition and vanishes at infinity.

"Pseudo-Radiation Conditions for Derivatives of Radiating Functions." J. of Math Analysis and its Applications 18, 3, (1967) pp. 561-564.

This paper introduces a new radiation condition satisfied by cartesian derivatives of a radiating solution to the reduced wave equation.

The function $g(x,y)$ and its derivatives are assumed to satisfy the differential Sommerfeld condition

$$\lim_{r \to \infty} \sqrt{r}\left(\frac{\partial g}{\partial r} - iKg\right) = 0$$

uniformly in θ, $a \leq \theta \leq b$ and certain boundedness conditions at infinity. Under these assumptions, it follows that

$$\lim_{r \to \infty} \int_{\alpha_o}^{\beta_o} f(\theta)L(D^{\alpha}g)(r,\theta)d\theta = 0, \qquad a \leq \alpha_o \leq \beta_o \leq b,$$

where the maximum value of α depends on the assumed degree of regularity of g. Here L represents the Sommerfeld rediation operator and $f(\theta)$ is an arbitrary function of θ having a bounded θ-derivative.

These papers are based on material included in EM-204 (Morgan), which is his doctoral dissertation.

III. Problems involving thin domains

The methods of the works in this category can be applied to a great variety of physical problems. In each case, some characteristic dimension in the problem is assumed to be proportional to a small parameter ε. Then solutions are sought which have asymptotic expansions in ε, for small ε. In EM-224 (Matkowsky), problems involving thin plates and shells, and eigenvalue problems for the Laplace operator involving thin domains are treated. In EM-226 (Handelsman and Keller), axially symmetric electrostatic and magnetostatic potentials around a slender body of revolution are found. The same methods are used in EM-228 (Geer and Keller) to describe two-dimensional potential flow around a thin airfoil, and the electrostatic field in the exterior of a thin conductor.

The following is a more detailed description of each of the preceding reports:

In EM-224 (Matkowsky), the problems associated with the asymptotic solution of partial differential equations in thin domains are considered. Fox and Westbrook have considered the asymptotic solution of potential problems in thin plates and shells of constant thickness. They showed that a series solution in powers of the thickness is a valid asymptotic representation of the solution. In EM-224, a more general class of fourth and second order elliptic boundary value problems in domains of variable thickness is treated. A systematic formal procedure for the determination of the various terms in the asymptotic solution is presented, and it is proven that the formal series is actually asymptotic to the exact solution of the problem. The method is apllied to the solution of two physical problems: 1) The three dimensional elec-

trostatic potential problem; 2) The torsion problem for cylindrical tubes. In section III, the asymptotic solution of the eigenvalue problem for the Laplace operator in thin two dimensional domains of variable thickness is considered. The domains considered are a) a thin ring-like domain bounded by two closed curves, and b) a section of the above ring.

In section IV, the eigenvalue problem for the Laplace operator in a two dimensional domain bounded by a closed convex curve is treated. The thin region is the region in which the amplitudes of the eigenfunctions are appreciably different from zero. This problem has been investigated by Keller and Rubinow, who considered it from the point of view of geometrical optics, constructing eigenfunctions by means of rays. The eigenfunctions which are obtained correspond, in acoustics, to the whispering gallery modes, by means of which a person speaking near the wall of a convex room, can be heard across the room near the wall, but not in the interior of the room. The results for these eigenfunctions and the corresponding eigenvalues provide improvements on the results of Keller and Rubinow.

In EM-226 (Handelsman and Keller) consider the classical problem of determining the electrostatic and magnetostatic potentials around a conducting body. A new method of solving them for slender bodies of revolution in axially symmetric external fields is found. It yields uniform asymptotic expansions of the potentials in terms of ε, the square of the slenderness ratio of the body which is the ratio of the body's maximum radius to its length. The method has been used before to determine the axially symmetric potential for flow about a slender body of revolution [1], which is also the magnetostatic potential

around a conducting body. Now it is used to obtain the electrostatic potential and then obtain the capacity and polarizability of the body.

The method consists in representing the potential as a superposition of potentials of point sources of unknown strengths distributed along a segment of the axis inside the body. The source strength satisfies a linear integral equation for which we obtain a uniform asymptotic solution. From it, the extent of the source distribution is found. The fact that the source distribution does not extend to the ends of the body permits the resulting asymptotic expansion to be uniform even at the ends. Both the cases of an isolated slender body and of a slender body in the presence of another axially symmetric body are treated. In addition the electrostatic potential for a uniform electric field applied perpendicularly to the axis of the body is obtained.

In EM-228 (Geer and Keller), the two dimensional irrotational flow of an incompressible inviscid fluid past a thin cylindrical airfoil and the two dimensional electrostatic field in the exterior of a thin cylindrical conductor are considered. The complete uniform asymptotic expansions of the two potentials are obtained with respect to ε, the slenderness ratio of the profile curve C of the cylinder. Only curves C which have axes of symmetry are treated. To find the symmetric part of the potential due to the presence of the body, it is represented as a superposition of potentials of point sources distributed along a segment of the axis inside C. The source strength distribution satisfies a linear integral equation. The uniform asymptotic expansion of the solution of this equation is obtained by adapting the method of Handelsman and Keller. The anti-symmetric part is obtained by using the

harmonic conjugate of an appropriate symmetric potential. Complete expansions of the virtual mass, the polarizability, and the lift of the cylinder are obtained.

IV. Propagation in Random Media

The properties of most media depend in a complicated way upon position and time from a molecular point of view, this is because the molecules constituting any medium have velocities, positions and possibly other properties which are distributed in a very complex manner. From a macroscopic point of view, this is often the case for fluids because they are in turbulent motion. For fluids in laminar motion there are variations in properties associated with thermal fluctuations. Solids are often mixtures of heterogencous substances or aggregates of tiny crystals with different orientations, etc. As a consequence of this variation of the properties of a medium, the propagation of waves in the medium is effected in many ways. The complexity of the variation of the properties makes it difficult to analyze these effects analytically in a direct way.

To overcome this difficulty, the mathematical concept of a random medium is introduced. This is just an ensemble of media, each designated by the value of a label α, together with a probability distribution $p(\alpha)$. The solution u of a wave propagation problem for the medium α depends upon α. Therefore it is denoted by $u(\alpha)$ and with it is associated the probability distribution $p(\alpha)$. This makes $u(\alpha)$ a random wave. Then the mean value of any functional $F[u]$ is denoted by $\langle F[u]\rangle$ and is defined by

$$\langle F[u]\rangle = \int F[u(\alpha)]p(\alpha)d\alpha. \tag{1}$$

It is assumed that certain statistical properties of the wave in the

random medium will correspond to properties of the wave in a given complex medium, provided that the random medium is chosen appropriately to model that medium.

In a general way the problem and the basic methods employed in reports EM-208, 210 and 231 may be formulated as follows. Let it be assumed that in the medium α, the state of the system $u(\alpha) = u(x,t;\alpha)$, a vector function of space and time,

$$L(\alpha)u = g \tag{2}$$

where $L(\alpha)$ is a linear operator depending on α, and g is a given non-random function. If $L(\alpha)$ is invertible, as we assume it to be for almost all α, we may write the solution of (1) as

$$u(\alpha) = L^{-1}(\alpha)g.$$

The average value $\langle u\rangle$ of $u(\alpha)$ is then seen to be

$$\langle u\rangle = \langle L^{-1}\rangle g, \tag{3}$$

or equivalently,

$$\langle L^{-1}\rangle^{-1} \langle u\rangle = g. \tag{4}$$

An important special case of (4) is the inhomogeneous one:

$$\langle L^{-1}\rangle^{-1} \langle u\rangle = 0. \tag{5}$$

$\langle L^{-1}\rangle$ is translationally invariant in a medium which as, we shall now assume, is statistically homogeneous. This implies that its improper eigenfunctions [eigenfunctions corresponding to the continuous spectrum] are exponential functions; specifically, of the form

$$\langle u\rangle = A \exp[i(k\cdot x-\omega t)] \tag{6}$$

where A, k, and ω are called the amplitude vector, the propagation vector and the angular frequency respectively. Inserting (6) into (5) we obtain the following linear algebraic system for the vector A

(7) $$\exp[-i(k\cdot x-\omega t)]\langle L^{-1}\rangle^{-1} \exp[ik\cdot x-\omega t]A = 0$$

The condition that this system have a non-trivial solution is the vanishing of the coefficient matrix:

(8) $$0 = \det[\exp[-i(k\cdot x-\omega t)]\langle L^{-1}\rangle^{-1}\exp[i(k\cdot x-\omega t)]]$$

To proceed further, it is necessary to evaluate the operator $\langle L^{-1}\rangle^{-1}$. This can be done in the case where L is the sum of a non-random operator M and a random part V

(9) $$L = M + V = M[1 + M^{-1}V]$$

V being so 'small' that the operator norm is less than unity. In this case L^{-1} has the Neumann stress expansion

(10) $$L^{-1} = \sum_{n=o}^{\infty} (-M^{-1}V)^n M^{-1}$$

and

(11) $$\langle L^{-1}\rangle^{-1} = M + \langle V\rangle + \langle V\rangle M^{-1}\langle V\rangle - \langle VM^{-1}V\rangle + O[(M^{-1}V)^3].$$

Inserting this expression for $\langle L^{-1}\rangle^{-1}$ into (8) we obtain a dispersion equation relating the angular frequency and propagation vector k of a time-harmonic plane wave. This dispersion relation involves only the mean values and the two point two time correlation functions of the random in homogeneities. From the dispersion equation the propagation velocity and attenuation coefficient of the wave can be obtained

In EM-210 (Keller), the theory sketched above is presented in

detail and applied to waves governed by linear differential equations. A smilar approach was used earlier by F.C. Karal and J.B. Keller in EM-208 for a special physical system [Effective Dielectric Constant, Permeability, and Conductivity of a Random Medium and the Velocity and Attenuation Coefficient of Coherent Waves]. The physical system was Maxwell's equations in which the dielectric constant, permeability and conductivity were considered random functions of position. The effective dielectric constant, permeability, and conductivity were defined as the appropriate proportionally factors between appropriate pairs of averaged fields, the average electric field, dielectric displacement, electric current etc., assuming as in the above discussion that the quantities are time-harmonic plane waves. In addition, the dispersion equation for the propagation constant of the average (coherent field) was analysed and solved for high and low frequency fields yielding the phase velocity and attenuation coefficient in these frequency ranges.

In a "Survey of the Theory of Wave Propagation in Continuous Random Media" [EM-233] presented at the Polytechnic Institute Symposium on Turbalence of Fluids and Plasma April 16-18, 1968, J.B. Keller recapitulates the theory presented above and then distringuishes between two kinds of methods that are employed in applying the theory to particular problems 'Honest' methods are direct mathematical procedures based upon explicitly formulated symplying hypotheses about the random medium 'Dishonest' methods are those which employ mathematical or physical assumptions about the solution. We discussed one such simplification earlier when we required that the random part of the operator L

be small in comparison to the non-random part [See equation (9)]. Other assumptions made about the medium are that it is Gaussian, Markoffian, or that it has some special statistical character.

Professor Keller first applies the general theory to the multiple scattering of scalar waves, after making the smallness assumption described above, and obtains the multiple scattering a Born expansion for the wave. Next, from this expansion, he shows how to determine the attenuation of the wave by an energy loss argument. Then, using a thin slab argument he shows how to derive both the attenuation and retardation of the wave. The same result is recovered once again by a selective summation of the multiple scattering series. These three methods are dishonest and yield results of limited accuracy. Next, the author describes an honest method which yields an exact equation satisfied by the mean wave. From this equation the effective propagation constant can be determined to any order in the perturbation parameter. To the lowest order it yields the results of the dishonest methods. The same honest method is then employed to obtain an equation satisfied by the second moment or two-point correlation function of the wave. Finally, the method is applied to the Navier Stokes equations to obtain a set of equations for the mean and two point correlation tensor of the velocity in an incompressible fluid in turbulent motion.

V. Miscellaneous

Report EM-231 (Frisch and Bowret) does not fall in any of the previous categories. It is concerned with the connection between quantum mechanical commutation relations and statistics. The results are applied to linear stochastic equations, such as arise in the study of turbulent convection, wave propagation in random media, impurity scattering, etc.

The generalized commutation relations $A_iB_j - \lambda B_jA_i = \Gamma_{ij}I$ are introduced, where A_i and B_i are adjoint of each other, I is the identity, and Γ_{ij} is a real covariance. For $\lambda = +1$ (resp. -1; resp. 0) the parastochastic function $A_i + B_i$ is given an interpretation in terms of Gaussian random functions (resp. a two-valued stationary Markov process; resp. infinite symmetric random matrices of the type considered by Wigner in connection with energy-levels of heavy nuclei). In the Fock-space realization, A_i and B_i appear as destruction and creation operators for bosons (resp. fermions; resp. boltzmannons, i.e. distinguishable particles). A few purely algebraic theorems are proved, which are applied to linear stochastic equations (equations with random coefficients). Existence and uniqueness being presupposed, mean Green's functions are shown to satisfy closed master-equations. A linear functional differential master-equation is obtained for equations with Gaussian coefficients. It is shown that the often-used first cumulant-discard closure assumption, which leads to a very simple master-equation, is exact for differential equations with a two-valued stationary Markovian coefficient. A parastochastic reformulation of the theory

of Kraichnan is given, and his nonlinear master-equations are for the first time rigorously derived without any recourse to perturbation theory or to diagrams. Kraichnan's random-coupling model is obtained by replacing scalar stochastic quantities by Wigner matrices or, equivalently, bosons by boltzmannons (i.e. changing λ from +1 to 0). Finally the nonlinear Kraichnan equation $dy(t)/dt = -\int_0^t y(t-t')\Gamma(t-t')y(t')dt'$ is reduced to a linear parastochastic equation in the Fock-space; existence, uniqueness, boundedness, and asymptotic behaviour are obtained.

"Steady Magnetogasdynamic Flow Past a Point Source" [J. Bazer and D. H.Yen]

It is a well-known fact of conventional gasdynamics that the disturbance arising from a perturbing point source P in a steady supersonic stream lies entirely within the Mach cone having its apex at P and lying downstream of this point. To obtain the field of P, one approach is to reduce the steady state problem in three space variables to an initial-value problem for a hyperbolic equation in two space variables. The third variable, measured along the direction of the free stream flow, plays the role of time. In EM-230 (J. Bazer and D.H.Y. Yen), attack the corresponding problem in steady magnetogasdynamic flow, the governing equations being the linearized Lundquist equations. The situation here is complicated by anisotropy and the presence of essentially three 'sound' speeds [the so called, fast, slow and Alfvén speeds]. As in gasdynamics when the free stream flow is fast enough specificaly, when the free-stream velocity exceeds fast disturbance speed in the direction opposite to that of the free stream - Lundquist's system of equation's is hyperbolic with respect to the direction of the free streams. In this case, it is shown that the magnetogasdynamic analog of the Mach cone

consistes of five distinguishable cones: a fast cone which envelopes the entire disturbance due to the point source; two degenerate Alfvén cones (lines) which carry delta-function like singularities, and two slow cones in which there is no disturbance at all. Explicit expressions [modulo the solution of a certain determinantal equation] are given for the field wherever excitation is present.

Apart from its intrinsic interest, the solution given by Bazer and Yen is a fundamental solution of governing system of equations. As such, it may, in principle, be used to solve various boundary-value, problems arising in connection with flow past bodies immerses in the medium.

Some Distinguished Guest Speakers at Electromagnetic Theory Seminars

P.L. Christiansen - Technological University of Denmark

L. Felsen - Brooklyn Polytechnical Institute

L.E. Fraenkel - Cambridge University

U. Frisch - Institut d'Astrophysique

S. Hong - M.I.T.

L. Howard - M.I.T.

S.T. Kuroda - Yale University

R. F. Miller - National Research Council of Canada

R. Newburgh - Air Force Cambridge Research Laboratories

H.M. Nussenzweig - U. of Rochester

N. Sauer - Pretoria, South Africa

Complete List of Reports and Publications

The following is a complete list of reports and publications of work done under Contract AF 19(628)3868 by the Division of Electromagnetic Research of the Courant Institute of Mathematical Sciences.

EM-194	R. M. Lewis J. B. Keller	Asymptotic Methods for Partial Differential Equations: The Reduced Wave Equation and Maxwell's Equation	1963
EM-195	J. Bazer L. Rubenfeld	Diffraction of Electromagnetic Waves by a Circular Aperture in an Infinitely Conducting Plane Screen Published: SIAM J. of Applied Math., 13,2, (1965), pp. 558-585.	1964
EM-196	R. M. Lewis	The Progressing Wave Formalism Published: Bklyn. Poly. Symposium on Quasi-Optics, June 1964.	1964
EM-197	R. M. Lewis	Asymptotic Methods for the Solution of Dispersive Hyperbolic Equations Published: Proc. of Wisconsin Symposium, 1964.	1964
EM-198	S. N. Karp F. C. Karal	Phenomenological Theory of Multi-Mode Surface Wave Excitation, Propagation, and Diffraction Published: Quart. Appl. Math., 24, 3, (1966), pp. 239-247.	1964
EM-199	S. N. Karp N. Zitron	A Plane Wave Expansion Theory for Cylindrically Radiated Fields	1964
EM-201	S. N. Karp F. C. Karal	Phenomenological Theory of Multi-Mode Surface Wave Structures Published: Bklyn. Poly. Symposium on Quasi-Optics, June 1964.	1964
EM-202	T. S. Chu R. G. Kouyournjian	On the Surface Wave Diffraction by a Wedge Published: IEEE Trans. on Antennas and Propagation, AP-13, 1, (1965), pp. 159-164.	1964

EM-203	H. M. Nussenzveig	High Frequency Scattering by an Impenetrable Sphere Published: Annals of Phys., 34, 1, (1965), pp. 23-95.	1964
EM-204	R. Morgan	Some Mathematical Problems on the Phenomenological Theory of Multi-Mode Surface Wave Diffraction	1964
EM-208	J. B. Keller	Effective Dielectric Constant Permeability and Conductivity of a Random Medium and the Velocity and Attenuation Coefficient of Coherent Waves Published: J. Math. Phys., 36, (1966), pp. 661-670.	1965
EM-210	J. B. Keller	The Velocity and Attenuation of Waves in a Random Medium Published: Proc. Interdisciplinary Conf. on EM Scattering II, 1967: Gordon and Breach, pp. 823-834.	1965
EM-212	R. Morgan	Uniqueness Theorem for a Multi-Mode Surface Wave Diffraction Problem Accepted: Quart. Appl. Math.	1965
EM-213	R. Morgan F. C. Karal S. N. Karp	Multi-Mode Surface Wave Diffraction by a Right-Angle Wedge Published: Quart. Appl. Math., 24, 3, (1966), pp. 263-266.	1965
EM-215	R. Morgan F. C. Karal S. N. Karp	Solution to the Phenomenological Problem of a Magnetic Line Source Above a Plane Structure that Supports N-Excited Surface Wave or Leaky Wave Modes Published: SIAM J. of Applied Math., 15, 6, (1967), pp. 1363-1377.	1965
EM-216	S. N. Karp F. C. Karal	Generalized Impedance Boundary Conditions with Application to Surface Wave Structures Published: Proc. of the URSI Symposium on Electromagnetic Wave Theory, Delft, The Netherlands, 1965, Pergamon Press, (1967).	1965
EM-217	D. Ludwig	Uniform Asymptotic Expansion of the Field Scattered by a Convex Object at High Frequencies Published: Comm. Pure & Appl. Math., 20, 1, (1967), pp. 103-138.	1966

EM-218	J. B. Keller	The Inverse Problem of Electromagnetic Scattering by a Metallic Object Published: Proc. of Conf. on Inverse Scattering Problems, MITRE Corp., 1, 1, (1965)	1965
EM-219	N. Bleistein R. M. Lewis D. Ludwig	Uniform Asymptotic Theory of Creeping Waves Published: Comm. Pure & Appl. Math., 20, 2, (1967), 295-328.	1966
EM-220	R. M. Lewis	Uniform Asymptotic Theory of Edge Diffraction Accepted: J. Math. Phys.	1966
EM-221	B. Rulf	On the Relation Between Creeping Waves and Lateral Waves on a Curved Interface Published: J. Math. Phys., 8, 9, (1967), pp. 1785-1793.	1966
EM-224	B. Matkowski	Asymptotic Solution of Partial Differential Equations in Thin Domains	1966
EM-225	B. Rulf	Asymptotic Expansions of the Field in the Penumbra of a Large Sphere Published: IEEE Antennas and Prop., AP-15, 5, (1967), pp. 704-705.	1966
EM-226	R. A. Handelsman J. B. Keller	The Electrostatic Field Around a Slender Conducting Body of Revolution Published: SIAM J. of Applied Math., 15, 4, (1967), pp. 824-841.	1966
EM-228	J. Geer J. B. Keller	Uniform Asymptotic Solutions for Potential Flow Around a Thin Airfoil and the Electrostatic Potential About a Thin Conductor Published: SIAM J. of Applied Math., 16, 1, (1968), pp. 75-101.	1967
EM-229	J. B. Keller H. Y. Yee L. B. Felsen	Ray Theory of Reflection from the Open End of a Wave Guide Published: SIAM J. of Applied Math., 16, 2, (1968), pp. 268-300.	1967
EM-230	J. Bazer D. H. Y. Yen	Steady Magnetogasdynamic Flow Past a Point Source Published: SIAM J. of Applied Math., 16, 4, (1968), pp. 808-833.	1967

EM-231	U. Frisch	Parastochastics	1967
EM-232	R. M. Lewis B. Granoff	Asymptotic Theory of Wave Propagation in an Inhomogeneous Anisotropic Plasma <u>Accepted</u>: Proc. of the URSI Symposium on Electromagnetic Theory, Stresa, Italy 1968.	1968
EM-233	J. B. Keller	A Survey of the Theory of Wave Propagation in Continuous Random Media <u>Accepted</u>: Proc. Symposium on Turbulence of Fluids & Plasmas, Bklyn. Polytechnic Institute, 1968.	1968

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13. ABSTRACT

This report is a summary of research activities pursued during the period from January 2, 1964 to January 1, 1969 by the Division of Electromagnetic Research of the Courant Institute of Mathematical Sciences at New York University, under Contract Number AF 19(628)3868 with the Air Force Cambridge Research Laboratories. The research program was a continuation of work done under Contract Number AF 19(604)5238 with the same agency.

The object of the research was to extend existing knowledge of classical electromagnetic theory with emphasis on electromagnetic wave-guiding radiation and communication, and diffraction problems.

Investigations have been conducted in various areas in accordance with these objectives and are explained in the body of this report.

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